WHAT ARE OTOB & dB?

by

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1 INTRODUCTION

For vibration analysts and testers, it is frequent to deal with data in units of "OTOB" and "dB". The purpose of this note is to provide a little help to clearly understand these units in addition to the enormous existing literatures that have described them. The acronym OTOB stands for one third octave band; and dB stands for decibel.

2 OTOB

An 'octave' is a frequency interval between any two frequencies having *a ratio of two*. It is called an octave from the music tradition where an octave spans eight notes of the scale. The second harmonic [2] of a spectral component is one octave above the fundamental. In acoustical measurements, sound pressure level is often measured in octave bands, and the center frequencies of these bands are defined by the ISO. Vibration measurements are seldom expressed as octave band levels, but the US Navy has used one third (1/3) octave band (OTOB) analysis for vibration measurements on submarines for a long time. Therefore, octave is defined as:

$$f_2 = 2^{octave} \cdot f_1$$

$$octave = \log(f_2 / f_1) / \log(2)$$

If ten increments of equal spacing[3] divide one major unit[4] in the 10-based logarithmic scale, what is the ratio of the higher value to the lower value of each sub-band increment? To answer this question, it is necessary to recall that the ratio of the higher value to the lower value of one major unit is always 10. Thus, if a major unit in logarithmic scale is evenly divided into ten sub-band increments, then the visual length Δ of each sub-band increment should be:

$$\Delta = \tfrac{1}{10} \cdot \log(10) = \log(10^{1/10})$$

Therefore, the ratio of the higher frequency to the lower frequency of each subdivision is:

$$f_2 / f_1 = 10^{1/10}$$

It follows that:

$$\log(f_2 / f_1) = 0.1 \approx 1/3 \cdot \log(2) = 0.1003433$$

As can be inferred from the above equation, each of the ten equally logarithmic spacing subdivisions is named as "One Third Octave Band".

It shall be noted that frequency bandwidths such as octave and one-third-octave bands are constant percentage bandwidths since the bandwidth is always a constant percentage of the center frequency. Thus the frequency bandwidth of each OTOB increases with its central frequency (see a typical table listed in Appendix at the end).

In the case of micro-gravity analysis of NASA projects, it is necessary to sub-divide each OTOB to supplement additional frequencies to *adequately capture off-peak responses*. It is required that the final frequency density be

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[2] A harmonic frequency is an integer multiple of the given frequency.

[3] It means a constant logarithmic spacing.
[4] It is such a unit as from one order of magnitude to the next higher or lower order of magnitude (e.g. from 100 (10^2) to 1000 (10^3) or 1 (10^0) to 0.1 (10^{-1}) and etc.).

sufficient to include at least one additional frequency within the half-power bandwidth of the modes. For example, NASA Document PIRN 57000-NA-110H [2] suggests sufficient logarithmic frequency spacing be used in each OTOB. It is natural to use evenly spaced frequency bands, as required typically for the use of test data. However, evenly spaced bandwidth should be less than or equal to the half-power bandwidth at the center frequency of the OTOB. If 0.5% damping ratio is assumed then each OTOB has to consist of a minimum of 23 sample bandwidths. The number of 23 is determined from the following formula:

$$N = \frac{\log(f_1/f_2)}{\log(\omega_1/\omega_2)} = \frac{\frac{1}{3} * \log(2)}{\log(1 + 2\cdot\zeta + 2\cdot\zeta^2 + O(\zeta^3))}$$

ζ is the damping ratio. Each of these evenly sub-divisions is called OSNOB[5] (One Sixty Ninth Octave Band).

3 dB

The unit of decibel (dB) is one tenth of the unit called Bell named after the famous Alexander Graham Bell[6]. The bell is a unit that denotes the magnitude of a quantity with respect to an arbitrary established reference value of the quantity, in terms of the logarithm (to the base 10) of the ratio of the quantities. One bell is equal to ten (10) decibels. The unit 'decibels' is first come from the study of sound where the sound intensity is the average rate of sound energy transmitted through a unit area normal to the wave direction at the point considered. This is a definition of power and may be expressed in watts per square meter. In acoustics, 10^{-12} w/m2 is chosen as reference value for sound power because it is near the lower limit of the human range of hearing. It is usual, however, to express power in decibels, dB, which is a term used to give the relative magnitude of two powers by comparing the one under consideration to a reference standard. Since power density with respect to frequency (g^2/Hz) is used as an measurement of vibration intensity in the PSD[7] of shuttle payload environmental testing, if decibel is used as an unit to describe this relative intensity, then the decibel shall be defined as:

$$dB = 10 \cdot \log(P_2 \ / \ P_1)$$

In acoustics, if sound-pressure is used as a measurement reference of relative intensity, then the constant 20 instead of 10 shall be used in the above definition (see pp. 264 of [3]). The reason behind this is try to make readings from two type measurements equivalent. Since log(2) = 0.30103, a "3 dB" difference means the power intensity of P2 is about twice of the power intensity of P1. Decibels do not add numerically as linear figures do; i.e., 70 dB + 70 dB = 73 dB since doubling power results in a 3-dB increase in sound pressure.

4 REFERENCE

[1] Kenneth McConnell, Vibration Testing, Theory and Practice, John Wiley & Sons, Inc. 1995.

[2] SSP 57000 Rev. E, "ISS Pressurized Payloads Interface Requirements" (PIRN NO: 57000-NA-0110H) 12/09/99

[3] Steven W. Smith, The Scientist and Engineer's Guide to Digital Signal Processing, 1997 by California Technical Publishing.

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5 Because 3 x 23 = 69.

6 As quoted from the presentation material prepared by Dr. Karen Dilka in his web site, quote "As the son and grandson of speech experts, he had a unique knowledge of the possibilities of sound. And as the son of a deaf mother, he had a true appreciation of the effort required to live in a hearing world. These two factors helped set Bell on the road to the telephone. The invention of the telephone made Bell great and famous." unquoted.

7 PSD stands for power spectral density.

5 APPENDIX

The following table shows an example of a typical OTOB division of frequency spectra:

1/10 decade	Lower	Central	Upper	Δω
	Hz			
-20	0.008913	**0.010000**	0.011220	0.002308
-19	0.011220	0.012589	0.014125	0.00290519
-18	0.014125	0.015849	0.017783	0.00365742
-17	0.017783	0.019953	0.022387	0.00460442
-16	0.022387	0.025119	0.028184	0.00579662
-15	0.028184	0.031623	0.035481	0.00729751
-14	0.035481	0.039811	0.044668	0.00918702
-13	0.044668	0.050119	0.056234	0.01156577
-12	0.056234	0.063096	0.070795	0.01456045
-11	0.070795	0.079433	0.089125	0.01833052
-10	0.089125	**0.100000**	0.112202	0.02307675
-9	0.112202	0.125893	0.141254	0.02905191
-8	0.141254	0.158489	0.177828	0.03657419
-7	0.177828	0.199526	0.223872	0.04604417
-6	0.223872	0.251189	0.281838	0.05796618
-5	0.281838	0.316228	0.354813	0.0729751
-4	0.354813	0.398107	0.446684	0.0918702
-3	0.446684	0.501187	0.562341	0.11565773
-2	0.562341	0.630957	0.707946	0.14560446
-1	0.707946	0.794328	0.891251	0.183305
0	0.891	**1.000**	1.122	0.231
1	1.122	1.259	1.413	0.291
2	1.413	1.585	1.778	0.366
3	1.778	1.995	2.239	0.460
4	2.239	2.512	2.818	0.580
5	2.818	3.162	3.548	0.730
6	3.548	3.981	4.467	0.919
7	4.467	5.012	5.623	1.157
8	5.623	6.310	7.079	1.456
9	7.079	7.943	8.913	1.833
10	8.913	**10.000**	11.220	2.308
11	11.220	12.589	14.125	2.905
12	14.125	15.849	17.783	3.657
13	17.783	19.953	22.387	4.604
14	22.387	25.119	28.184	5.797
15	28.184	31.623	35.481	7.298
16	35.481	39.811	44.668	9.187
17	44.668	50.119	56.234	11.566
18	56.234	63.096	70.795	14.560
19	70.795	79.433	89.125	18.331

1/10 decade	Lower	Central	Upper	Δω
	Hz			
20	89.125	**100.000**	112.202	23.077
21	112.2	125.9	141.3	29.1
22	141.3	158.5	177.8	36.6
23	177.8	199.5	223.9	46.0
24	223.9	251.2	281.8	58.0
25	281.8	316.2	354.8	73.0
26	354.8	398.1	446.7	91.9
27	446.7	501.2	562.3	115.7
28	562.3	631.0	707.9	145.6
29	707.9	794.3	891.3	183.3
30	891.3	**1000.0**	1122.0	230.8
31	1122.0	1258.9	1412.5	290.5
32	1412.5	1584.9	1778.3	365.7
33	1778.3	1995.3	2238.7	460.4
34	2238.7	2511.9	2818.4	579.7
35	2818.4	3162.3	3548.1	729.8
36	3548.1	3981.1	4466.8	918.7
37	4466.8	5011.9	5623.4	1156.6
38	5623.4	6309.6	7079.5	1456.0
39	7079.5	7943.3	8912.5	1833.1
40	8912.5	**10000.0**	11220.2	2307.7
41	11220.2	12589.3	14125.4	2905.2
42	14125.4	15848.9	17782.8	3657.4