A Simplified Simulation on Loading Distribution at Bolt Threads

by

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1. INTRODUCTION

The strength assessment of a bolt should include a verification of the shear strength of bolt threads against the thread stripping. In the past, this was usually done by comparing an average shear stress across the cross sections of all the threads that are engaged with its matting part. This average stress is the ratio of the total tensile force in the bolt shaft to the shear area of all the engaged threads. The shear area is approximately the cylindrical surface at pitch diameter times the number of engaged threads. A factor of safety is imposed on this average stress to account for any uncertainty, but the strength assessment based on this criterion becomes unacceptable. Most mechanical engineers recognize that when a bolt is in tension, more than 80% of the tensile force in bolt shaft is carried by the first few engaged threads. In this note, a very simple math model is proposed to estimate this phenomenon qualitatively and quantitatively.

2. NOMENCLATURE

A_s	cross sectional area at bolt shank
A_t	listed tensile area of bolt
A_{th}	cross sectional area between the bolt shaft and one revolution of thread
c	half height of beam cross section as defined at Fig.28 (pp.46 of [7])
D_c	largest diameter at thread crest
D_r	smallest diameter at thread root
D_p	pitch diameter of bolt thread
D_s	shank diameter
E	Young's modulus of bolt material
f_c	flat width at crest
f_r	flat width at root
G	Shear modulus of bolt material
h	thread height
I_t	area moment of inertia of one pitch thread
k_i	spring rates ($i = 1, 2, 3$)
l_s	shank length
p	bolt thread pitch
U_t	total potential energy
U_s	shear strain energy at thread
V	shear force at beam cross section
w	distribution force (force per length)
ν	Poisson's ratio of bolt material
x_i	displacement at node "i"

3. SIMPLE MODEL FORMULATION

In this simplified simulation, the bolt model consist of a number of spring-type elements. These springs simulate the elastic stiffness of various parts of a bolt. They are:

1) Spring for the part from bolt head to the 1st engaged thread,
2) Springs for the bolt shaft between neighboring threads, and
3) Springs to simulate each *pitch thread*[2] that is in engagement.

The proposed simple math model is shown in Figure 1. Springs k_1, k_2 and k_3 represent respectively the shank part, shaft part between threads and each pitch thread of the bolt. All threads are of same elastic stiffness.

The hatched areas shown in Figure 1 represent the contact surfaces of the mating parts of the bolt (e.g. nut or insert threads). They are assumed to be rigidly fixed. The deformation of the mating parts is not considered in the current simplified simulation.

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[2] One ***pitch thread*** is defined as one revolution of the thread that advances one pitch along the bolt shaft.

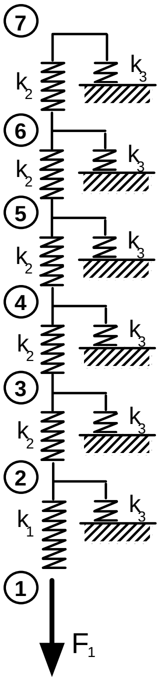

Figure 1

In this simplified model, seven node locations and twelve spring elements are used to model six engaged threads. For each node, there is only one degree of freedom (DOF) to represent the axial displacement x_i along the bolt longitudinal direction when the bolt is subjected to an external applied tensile force. We use the Principle of Minimum Potential Energy (see pp. 330 of [4]) to derive the static equilibrium equation. The total potential energy of this static system is:

$$U_t = \frac{1}{2}\ k_1\ (x_1 - x_2)^2\ + \{\frac{1}{2}\ k_2\ (x_3 - x_2)^2\ + \frac{1}{2}\ k_1\ (x_4 - x_3)^2\ + \cdots\ + \frac{1}{2}\ k_1\ (x_7 - x_6)^2\,\} + \frac{1}{2}\ k_3\ (x_2^2 + x_3^2 + \cdots + x_7^2) - F_1 \cdot x_1 \qquad \text{(3-1)}$$

The total potential energy should be a minimum to be in equilibrium. This leads to the following necessary conditions:

$$\frac{\partial\, U_t}{\partial\, x_i} = 0\ \ where\ i = 1, 2, \ldots, 7 \qquad \text{(3-2)}$$

Carrying out equation (3-2) will generate the following system of algebraic equations:

$$\frac{\partial\, U_t}{\partial\, x_1} = 0 \xrightarrow{yields} k_1\ (x_1 - x_2) = F_1$$

$$\frac{\partial\, U_t}{\partial\, x_2} = 0 \xrightarrow{yields} -k_1(x_1 - x_2) + k_2(x_2 - x_3) + k_3 x_2 = 0$$

$$\frac{\partial\, U_t}{\partial\, x_3} = 0 \xrightarrow{yields} -k_2(x_3 - x_3) + k_2(x_3 - x_4) + k_3 x_3 = 0$$

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$$\frac{\partial\, U_t}{\partial\, x_7} = 0 \xrightarrow{yields} -k_2(x_6 - x_7) + k_3 x_7 = 0$$

The above system of algebraic equations in a matrix equation will be:

$$\begin{bmatrix} k_1 & -k_1 & 0 & \cdots & 0 & 0 \\ -k_1 & k_1 + k_2 + k_3 & -k_2 & & & 0 \\ 0 & -k_2 & 2k_2 + k_3 & & & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & & & & 2k_2 + k_3 & -k_2 \\ 0 & 0 & & 0 & -k_2 & k_2 + k_3 \end{bmatrix} \cdot\cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_6 \\ x_7 \end{pmatrix} = \begin{pmatrix} F_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \qquad \text{(3-3)}$$

If all spring rates k_i and the applied loading F_1 are known, the displacements x_i at seven nodes can be solved from equation (3-3). The reaction force on each engaged thread can be calculated as:

$$F_{pt_j} = k_3 \cdot x_j\ \ where\ j = 2, 3, \cdots, 7 \qquad \text{(3-4)}$$

From the results of equation (3-4), calculate the percentage of total loading for each bolt thread.

The spring rates k_1 and k_2 of the shank and shaft between threads are based on the simple bar formula in tension:

$$k_1 = \frac{E \cdot A_s}{\ell_s} \qquad \text{(3-5)}$$

$$k_2 = \frac{E \cdot A_t}{p} \qquad \text{(3-6)}$$

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The bolt thread seems like a helical bar wrapped and welded around a cylindrical shaft. A longitudinal cross sectional view (see Figure 2) of the bolt shows that the thread part looks like many '*teeth*' being implanted along the bolt shaft surface.

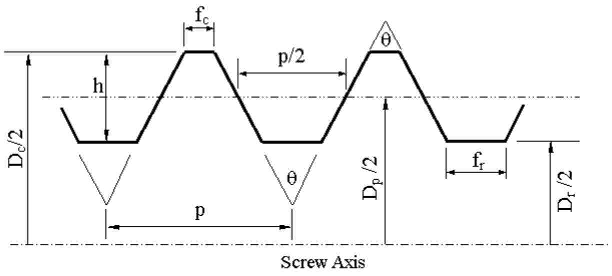

Figure 2

Based on this observation, the simplified simulation model approximates each bolt *pitch thread* as a *planar* ring of a symmetric trapezoidal cross section that is welded on a cylindrical surface. Figure 3 depicts a small circumferential increment of this ring. It also shows both the geometric data of bolt thread and loading condition that will be used to obtain the spring rates k_3 based on three different approaches.

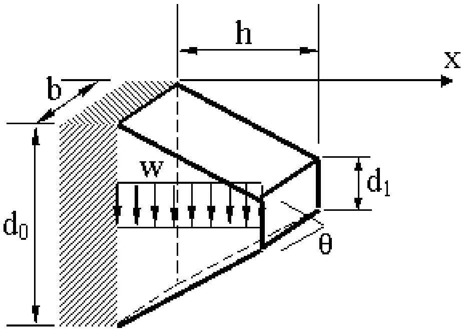

Figure 3

Because of the small ratio of the thread height to tooth-width (i.e. h/d_0), the deflection of bolt thread is dominated by shearing effect rather than the flexural bending. The first two methods assume the thread is of uniform rectangular cross section of constant depth d_0. Two existing formulae in the literature are modified to calculate the spring rate k_3 of each pitch thread. The first approximation of k_3 is based on the second term of equation (34) of [7] at pp. 49:

$$k_3 = \frac{24}{5}\,\frac{E \cdot I_t}{h^3} \cdot \left(\frac{5}{12}\frac{4h^2}{d_0^2} \cdot \frac{10}{8 + 5\upsilon}\right)$$

(3-7)

where

$$I_t = \frac{1}{12}\,\pi D_r\, d_0^3$$

The second approximation of k_3 is based on the fourth equation at pp. 202 of [8].

$$k_3 = \frac{5}{3}\left(\frac{A_{th} \cdot G}{h}\right)$$

(3-8)

where

$$A_{th} = \pi D_r\, d_0$$

(3-9)

The third approximation of k_3 is to model the thread as a linearly tapered bar of rectangular cross section as shown in Figure 3. The methodology is to use the shear strain energy in the deformed thread to find k_3. If U_s is the strain energy stored in a *linear* spring of spring rate k when it is stretched with a force F, the spring rate k can be calculated as:

$$k = \frac{F^2}{2 \cdot U_s}$$

(3-10)

The strain energy of a tapper block, such as the one shown in Figure 3, due to the transverse shear force $V(x)$ is[3]:

$$U_s = \int_0^h \frac{3\,V^2(x)}{5G\,A(x)}\,dx$$

(3-11)

where

$$V(x) = w \cdot (h - x)$$

$$A(x) = b \cdot (d_0 - \beta x)$$

$$\beta = 2 \cdot \tan\frac{\theta}{2}, \qquad and$$

$$h = \frac{d_0 - d_1}{\beta}$$

(3-12)

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[3] See Table 8.2 at pp. 150 of [9].

Integration[4] (3-10), we get:

$$U_s = \frac{3hw^2}{5Gb}\left\{\left[\frac{2h}{\beta}+\frac{d_1-3d_0}{2\beta^2}\right] + \left[\frac{h}{\beta}-\frac{d_0(d_0-2d_1)}{\beta^2(d_0-d_1)}\right]\cdot\log\left(\frac{d_0}{d_1}\right)\right\} \tag{3-13}$$

The item of *hw* is the total force *F* exerted on one pitch thread. By (3-10), the k_3 will be calculated as follows:

$$k_3 = \frac{5Gh(\pi D_r)}{6}\left\{\left[\frac{2h}{\beta}+\frac{d_1-3d_0}{2\beta^2}\right] + \left[\frac{h}{\beta}-\frac{d_0(d_0-2d_1)}{\beta^2(d_0-d_1)}\right]\cdot\log\left(\frac{d_0}{d_1}\right)\right\}^{-1} \tag{3-14}$$

The width *b* in (3-13) is substituted with one circumferential length (πD_r) of the thread circle at its root.

4. NUMERICAL SIMULATION

The bolt of NAS6703HU10 .1900"-32 CRES is used for the numerical simulation. The MathCAD algorithm is shown below:

Material Properties:

Young's Modulus, Poison's Ratio and Shear Modulus

$$E := 29000\,\text{ksi}$$

$$\nu := 0.29$$

$$G := \frac{E}{2\cdot(1+\nu)} = 11240\,\text{ksi}$$

Geometry Dimensions

Nominal Diameter	$D := 0.190\,\text{in}$
Pitch Diameter	$D_p := D - 0.649519\frac{1}{24}\cdot\text{in} = 0.1629\,\text{in}$
Minor Diameter	$D_r := 0.1528\,\text{in}$
Shank Diameter	$D_s := 0.190\,\text{in}$
Thread Pitch	$p := \frac{1\cdot\text{in}}{32}$

The spring rates k_1 and k_2 are:

$$k_1 := \frac{E\cdot A_s}{2D} \qquad k_1 = 2.152\times 10^{6}\cdot\frac{\text{lbf}}{\text{in}}$$

$$k_2 := \frac{E\cdot A_t}{p} \qquad k_2 = 1.856\times 10^{7}\cdot\frac{\text{lbf}}{\text{in}}$$

The length from bolt head to the 1st engaged thread is assumed to be twice the nominal diameter. The spring rates k_3 calculated per formulae (3-7), (3-8) and (3-14) are:

$$k3_1 := \frac{24}{5}\cdot\frac{E\cdot I_t}{h^3}\cdot\left(\frac{5}{12}\cdot\frac{4\cdot h^2}{{d_0}^2}\cdot\frac{10}{8+5\cdot\nu}\right) = 1.361\times 10^{7}\,\frac{\text{lbf}}{\text{in}}$$

$$k3_2 := \frac{2}{1}\cdot\frac{5}{6}\cdot\left(\left(\frac{A_{th}\cdot G}{h}\right)\right) = 1.246\times 10^{7}\cdot\frac{\text{lbf}}{\text{in}}$$

and

$$k3_3 := \frac{5}{6}\cdot G\cdot h\cdot\pi\cdot D_r\cdot\left[\left(\frac{2\cdot h}{\beta}+\frac{d_1-3d_0}{2\cdot\beta^2}\right)\ \ldots \atop +\left[\frac{h}{\beta}-\frac{d_0\cdot(d_0-2\cdot d_1)}{\beta^2\cdot(d_0-d_1)}\right]\cdot\log\left(\frac{d_0}{d_1}\right)\right]^{-1}$$

$$k3_3 = 1.568\times 10^{7}\,\frac{\text{lbf}}{\text{in}}$$

The loading distributions (in percentage of total applied force) along the 6 engaged threads for all the three approximate spring rates k_3 are shown in the following table and chart:

Thread No.	**Loading Distribution Percentage (%)**		
	Method - I	**Method - II**	**Method - III**
1	56.5	55.0	58.9
2	24.6	24.8	24.2
3	10.7	11.2	10.0
4	4.7	5.1	4.1
5	2.2	2.5	1.8
6	1.3	1.5	1.0

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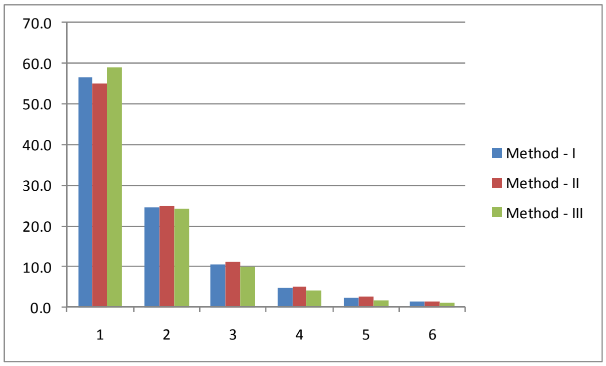
[4] Use the integration formulae in pp. 6 of [10].



To see the effects of spring rate k_3 on the loading distribution on bolt threads, ten evenly distributed k_3 (ranging from 1.0E+07 lbf/in to 3.0E+07 lbf/in) are analyzed by the same math model. Its results are shown in Figure 4.

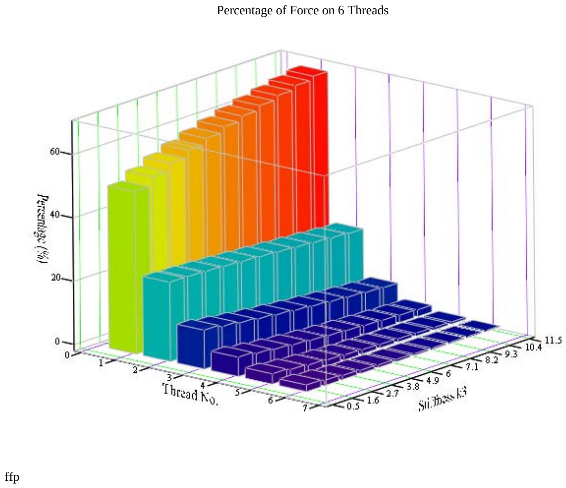


Figure 4

Figure 4 shows the first bolt thread usually take a 50% to 60% share of the total tensile force. The loading on the second bolt thread drops dramatically to about 20% of the total tensile force.

5. CONCLUSIONS

In reference [3] (pp. 593), it was stated that one of the means to improve bolt performance without increasing the nominal bolt diameter is to make load distribution on the threads more uniform. But how is this accomplished? One of the solutions is to make the first few engaged threads 'softer' than the later ones so that more loading can be passed to more threads to share the burden. It is simple in principle but very challenging from the standpoint of the manufacturing process and material metallurgy.

When the margin of safety is marginal by using classical average shear stress on all engaged threads, it is necessary to reevaluate the shear strength adequacy of the first thread based on the assumption that 60% of the force is carried by it. Since the interaction between the bolt threads and their mating part is a complex phenomenon, the analysis results from a linear elastic math model cannot give a sufficiently accurate prediction on thread loading. Especially when the total tensile force of the bolt is so high that the first few bolt threads may have been yielded into the material plastic zone. The analysis results indicate that it is highly possible that some local regions of the first one or two threads may often be yielded and consequently plastically softened in reality. This plastic softness will accordingly lower the spring rate k_3 in these threads and redistribute more loading to the other engaged threads; and make the loading distribution more uniform. Therefore, to reduce the risk, used bolts should be replaced. This should be the standard practice for those bolts used in safety-critical areas.

6. REFERENCE

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